Divergent Expectations:
Its Effect on Price Formation in an Equity Market

Robert A. Schwartz, Jian Hua, Deniz Ozenbas, and Jun Wang

Robert A. Schwartz is the Marvin M. Speiser Professor of Finance and University Distinguished Professor in the Zicklin School of Business, Baruch College, CUNY in New York, NY
robert.schwartz@baruch.cuny.edu
One Bernard Baruch Way, Box B10-225, New York, NY 10010
(646) 312-3467

Jian Hua is a Professor of Finance in the Zicklin School of Business, Baruch College, CUNY in New York, NY
jian.hua@baruch.cuny.edu
One Bernard Baruch Way, Box B10-225, New York, NY 10010
(646) 312-3487

Deniz Ozenbas is a Professor of Finance in the Feliciano School of Business, Montclair State University in Montclair, NJ
ozenbasd@montclair.edu
1 Normal Avenue, Montclair, NJ 07043
(973) 655-7522

Jun Wang is a Professor of Economics in the Zicklin School of Business, Baruch College, CUNY in New York, NY
jun.wang@baruch.cuny.edu
One Bernard Baruch Way, Box B10-225, New York, NY 10010
(646) 312-3507

December 5, 2024

Abstract

Important academic analyses of price formation in the equity markets are based on the assumption that participants have homogeneous expectations. Relaxing this assumption, we deal with the reality that, because information sets are typically large and complex, investors have divergent expectations. In a divergent expectations environment, price discovery is a dynamic, complex, noisy process that involves elevated short period price volatility and return autocorrelations of first and higher orders. In the dynamic environment of divergent expectations and noisy price discovery, market structure matters, behavioral economics can have a role to play, and technical analysis can be valid.

Keywords: Efficient markets hypothesis, random walk, trading, technical analysis, divergent expectations, behavioral economics.

JEL Codes: G11, G12

Divergent Expectations:
Its Effect on Price Formation in an Equity Market

Important academic analyses of price formation in the equity markets, including the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT),[1] are based on the assumption that participants have homogeneous expectations. This simplification of the real world is required for the risk-based models to be built. The assumption of homogeneous expectations has also been accepted by those academicians who believe that rational share assessments based on identical information will lead to identical share valuations. We contend that this thinking is an oversimplification of reality.

Information sets can be enormous, complicated, incomplete, contradictory, misleading, and they can contain conflicting signals (e.g., profits are up but sales are down). Therefore, information sets are subject to different interpretations and to fuzzy valuations (i.e., the impossibility of directly translating complex information into share values with penny point precision). Accordingly, we expect that participants observing the same information will form different assessments of share values.

The homogeneity assumption precludes addressing three marketplace realities: (1) investors have divergent expectations, (2) investors have adaptive valuations (i.e. they reassess their own estimates upon learning the evaluations of others), and (3) investors have fuzzy evaluations.

The first reality, that investor expectations are divergent, has implications for a number of major issues. These include acknowledging that price discovery is the fundamental economic function of a stock exchange.[2] Additionally, it is important to focus on the challenges that a divergent expectations environment presents to traders, to exchanges, and to government regulators seeking to enhance the efficiency of market structure (e.g., the development of a

[1] Other models that assume homogenous expectation include Modern Portfolio Theory and the Rational Expectations Models. Black-Scholes Option Pricing Model also assumes that markets are frictionless, and all investors have access to the same information and can trade freely without restrictions.

[2] During his tenure (1976-1984) as CEO of the New York Stock Exchange, Batten participated in a meeting (which Schwartz attended) with a small number of academicians. Batten made one statement during the discussions concerning identifying what the fundamental function of a stock exchange is. His succinct comment was totally memorable: “We produce the price.”

National Market System that was initially called for by the Congressional Securities Acts Amendments of 1975). Liquidity (or the lack thereof) also matters in the non-frictionless world of divergent expectations.

Price discovery in a divergent expectations environment involves elevated short period (e.g., intra-day) price volatility as well as returns autocorrelations of first and higher orders. While our discussion is based on rational economic analysis, recognition of this opens the door for behavioral economics to play a role in understanding how price discovery operates. We further suggest that, in the dynamic environment of divergent expectations and noisy price discovery, technical analysis, which is being widely used by participants (especially on an intra-day basis with the use of various trading algorithms), can be valid. Nevertheless, technical analysis has been widely frowned upon by academicians.

The second marketplace reality, that investors have adaptive valuations, follows from divergent expectations. When individual expectations are divergent, investors can respond in a variety of ways to each other's share valuations that they learn about through direct person-to-person contact, from advice given by their brokers, and via public broadcasts and social media. Investors also respond to the prices that are being set in the marketplace, which is the response we focus on in this paper.

Fuzzy valuations, the third marketplace reality, is due to the complexity of fundamental information. Because of the large and complex information sets that they are dealing with, investors' evaluations cannot be made with penny point precision.

CAPM, a cornerstone of modern portfolio theory, shows the equilibrium configuration of prices that would be obtained in a frictionless, informationally efficient environment. CAPM, however, does not address the institutional arrangements that handle how prices are actually set in a non-frictionless, divergent expectations environment that characterizes a real-world marketplace. CAPM has no implications for trading or for the market structure that is required for transactions to be made and prices to be discovered. Thus, for understanding how fundamental information that underlies share values gets translated into security prices, CAPM, although an important model, is an incomplete model.

The Efficient Markets Hypothesis (EMH) holds when equity prices reflect all outstanding public information and thus additional risk adjusted returns cannot be realized by exploiting the information set. Random walk is a major test of the EMH. When *all* existing and anticipated information is reflected in share prices, only new and unexpected information can cause price changes and, thus, returns will not be autocorrelated, stock prices will follow random walks, and the EMH will prevail. Empirical analyses of individual stock return autocorrelations have mainly focused on daily returns and first order autocorrelation, and have been generally supportive of the EMH without taking account of the higher orders of autocorrelation.[3] But intra-day time periods, and higher orders of autocorrelation have not, our knowledge, been taken account of.

As we report in Section 2, volatility analysis, when applied to short period returns, shows that share prices do exhibit intra-day autocorrelations of first and higher orders. The cause of the autocorrelations is that, when investors have divergent expectations, price discovery is a complex, dynamic process that works itself out over time and, as it does, higher orders of autocorrelation are introduced.[4]

Focusing on divergent expectations and noisy price discovery builds a bridge between frictionless market formulations and a real-world marketplace. Recognizing that market produced prices do not follow random walks and, thus, that markets are not as efficient as the EMH implies, opens the door to three issues that should be further addressed: behavioral analysis, technical analysis, and the design and regulation of market structure.

In the first section of the paper, we focus on a conceptual framework for analyzing price discovery in a divergent expectations environment. In the second section, we turn to empirical analysis of inflated volatility and its cause: autocorrelations of first and higher order. The third section focuses on trading, behavioral economics and technical analysis. The fourth section deals with the effect that market structure (and its regulation) has on the quality of price discovery. In the concluding section, we suggest that CAPM and the realities of a non-frictionless, divergent expectations marketplace both have a role to play in understanding the operations of an equity market.

Section 1: Price Discovery in a Divergent Expectations Environment

Handa, Schwartz, and Tiwari (2003, HST) considers price determination in a divergent expectations environment that, for simplicity, is structured by sorting investors in one of two sets according to their expectations, and by letting their reservation prices to buy or to sell shares depict

---

[3] Lewellen (2022), provides a review of this literature in a working paper titled “Autocorrelation of stock and bond returns, 1960–2019.”

[4] See Hua and Schwartz (2024).

their expectations. In each of the two sets, expectations are homogeneous, while they are divergant between the sets. Investors in the bullish set have a high reservation price to buy, V_H, and those in the bearish set have a low reservation price to sell, V_L. Given the disparity between the reservation prices that differentiate between the two sets, a trade between any two participants (one a bull and one a bear) at a price within the range V_H and V_L will benefit both of the counterparties.

HST shows that a V_H participant will place a bid of P_B, that a V_L participant will place an offer of P_A, and that $V_H > P_A > P_B > V_L$ where the market bid-ask spread is $P_A - P_B$. HST assumes that all participants know V_H, V_L, the percentage, k, of participants who have a valuation of V_H, and know how others will respond to posted bid and offer quotes.

Let V_H = \$54, V_L = \$50, and $(P_A - P_B)/2$ = \$51.52. Because of their inability to assess share value with penny point precision, each trader's reservation price is a round number (e.g., \$50 or \$54). If a trade were to occur at, for example, the mid-spread price of \$51.52, each party would benefit because \$51.52 is below the buyer's reservation price and is above the seller's reservation price. With fuzzy valuations, the penny price precision of the transaction price (51.52) is attributable to an interaction (which could be viewed as an implicit negotiation) between the buyer and the seller that depends on the value of k. The price of \$51.52 is closer to V_L than to V_H because, for this illustration, we have taken k to be greater than 0.5, thereby causing competition between the buyers to drive price down because it is more intense than competition between the sellers.

Schwartz, Paroush, and Wolf (SPW, 2010) relax HST's simplifying assumption that participants know k but infer it from the buy and sell orders sent to the market. Operating in the context of HST's V_H, V_L, framework, SPW shows that price discovery (and its companion k discovery) precedes as trading progresses.

SPW introduces another factor that comes into play in a divergent expectations environment when investors do not know the value of k: they can have adaptive valuations. That is, a participant who receives some new information and has the initial, independent valuation of V_L may rethink his or her initial valuation and become a V_H buyer after observing that the preponderance of orders being delivered to the market are driving price up.

As SPW notes, adaptive valuations can be based on rational economic behavior because decision makers can respond to what Surowiecki (2005) refers to as the "wisdom of the crowd." Crowd wisdom was established by an eighteenth-century mathematician, Condorcet (1785), who showed that, when the members of a crowd make their decisions independently of each other, the average of their decisions will be more accurate than the decisions they have made individually. More recently, Ladha (1995) has shown that Condorcet's proof holds even in the presence of slight interdependence among the decision makers. In our context, the "individual decisions" are the individual assessments of share value, and a share's price in the marketplace is the broad market's assessment that has been realized through trading.

Hua and Schwartz (2024) present a variant of SPW's formulation and provide empirical evidence of accentuated volatility in daily returns that is caused by prices overshooting equilibrium values, pivoting, and reverting back in a divergent expectation, adaptive valuation environment where k is not known before trading starts. Given the patterns that dynamic, noisy price discovery can generate, an analysis of past price movements can provide traders with guidance in pricing and timing their orders, and technical analysis can help with this.

Davis and Schwartz (2021) consider the coexistence of fundamental and technical analysis with reference to two Greek philosophers, Plato and Aristotle. Their discussion, which focuses on Plato's analogy of a cave, equates Plato's thinking with fundamental analysis, and equates Aristotle's focus on observable behavior with technical analysis. In the analogy of the cave, truth cannot be observed directly but only as shadows cast on the walls of a cave. Fundamental analysis involves studying the shadows and inferring underlying truth from them. When the shadows are complex, Platonian observers will differ in their assessments, and Aristotelian observers will draw their own conclusions based on what they observe others doing in the marketplace.

Different observers of the shadows on the walls of Plato's cave interpreting the shadows differently is analogous to different stock analysts having divergent expectations about share values because of the complexity of information sets. Price discovery in a divergent expectations environment involves runs, reversals and other gyrations and, accordingly, Aristotelian thinking in the form of technical analysis enters the picture.

Schwartz, Paroush and Wolf, and Hua and Schwartz, do not take the Platonian fundamentalists and the Aristotelian empiricists to be two different sets of people. Rather, they assume that all individuals make their individual assessments of share value (Platonian behavior) and can adjust their price valuations upon observing how others are trading (Aristotelian behavior). Hua and Schwartz structure an individual's share valuation as a weighted combination of their own assessment and of the market's assessment.

For Condorcet's proof of market wisdom, participant assessments must be made independently, and thus the weight a participant places on the market's assessment in Hua and Schwartz's analysis largely depends on the individual's perception of the independence with which participant valuations are in fact being made. As Hua and Schwartz explain, if the perception of independence is sufficiently high, the weight given to the market's valuation can be relatively high and a momentum price move can start. But, as the stock's price continues to rise (or fall), the perception of independence can decrease, and the weight placed on the market's assessment will fall. As it does, it is clear that the stock's price has overshot its equilibrium value, and the price increase (decrease) is followed by a price decrease (increase). This dynamic process introduces higher order of autocorrelation and accentuates short period volatility.

Section 2: Accentuated Volatility, Returns Autocorrelation, and The Demise of Random Walk

To get evidence of price discovery's impact on the dynamic behavior of share prices, Hua and Schwartz (2024), recognizing the complexity of dealing with intraday data, base their empirical analysis on a contrast between daily and monthly returns. However, much price discovery occurs intraday, most predominantly in the first half hour of trading. Accordingly, in this paper, we obtain both a variance ratio (VR) and first order autocorrelation (CORR) to detect noise in half-hour returns.

To obtain the variance ratio, write

(1) $R_L = \sum_{s=1}^{T} R_S$

where R_L is the log of 1 plus the long interval return, R_S is the log of 1 plus the short interval return, and T is number of short intervals in a long interval. Taking the variance of both sides of (1) gives

(2) $Var(R_L) = T * Var(R_S) + 2 * \sum_{s=1}^{T-1} Cov\left(R_{S,1}, R_{S,1+s}\right)$

Rearranging gives the variance ratio,

(3) $\frac{T * Var(R_S)}{Var(R_L)} = 1 - 2\frac{\sum_{s=1}^{T-1} Cov\left(R_{S,1}, R_{S,1+s}\right)}{Var(R_L)}$

where Cov($R_{S,1}$, $R_{S,1+s}$) is the autocovariance in short period returns separated by "s" periods. Note that the variance ratio captures the full set of first and higher orders of returns autocorrelation.

From Equation (3) it follows that the variance ratio, $T^*Var(R_S)/Var(R_L)$, is greater than, equal to, or less than 1.00 if the covariance terms are, respectively, negative, zero, or positive.

Because VR reflects the impact of a broader set of primarily intra-day factors (including the bid-ask spread, liquidity trading and market impact), VR greater than 1.0 is not evidence of price discovery noise per se. We infer the importance of price discovery as a causal factor by running a multifactor regression where the independent variables could be related to price discovery noise but, most likely, not to the other causal factors.

Our study includes all common stocks traded on the NYSE and NASDAQ from January 1993 through December 2021. Intraday returns and daily spreads are calculated using NYSE Trade and Quote (TAQ) data. Firm-level stock data are from CRSP.

Employing half-hour and monthly return data, we assess intra-day variance with the variance ratio (VR) set forth in Equation 3. Using a rolling window of the preceding three years, we obtain VR by dividing the "monthualized" standard deviation of all intraday, 30-minute returns by the monthly return standard deviation. The stock specific explanatory variables include: the log of end of month market capitalization (*LNME*), order flow consolidation (market share) for each exchange (NYSE or Nasdaq), the monthly average of the proportion of the first half hour shares to the daily number of shares (*1st HH*), the monthly average of daily equal-weighted effective spreads obtained from WRDS's intraday indicators (*Spread*), and the monthly measure of daily return standard deviations (*Volatility*).

Summary statistics for our variables are presented in Table 1 for NYSE stocks (Panel A), and for Nasdaq stocks (Panel B). For NYSE stocks, the half-hour to one month returns VR has an average mean of 1.29, a median of 0.96, and a standard deviation of 2.14. Monthly first order daily returns autocorrelation (CORR) has an average of -0.06, a median of -0.07, and a standard deviation of 0.23. For Nasdaq stocks, VR has a higher average at 1.75, a median of 1.21, and a standard deviation of 2.25. CORR for Nasdaq has an average of -0.14, a median of -0.15, and a standard deviation of 0.24.[5]

First order autocorrelation is negative for 61% and 71% of NYSE and Nasdaq stock-months, respectively, while VR is greater than 1.00 for 71% and 79% of NYSE stocks and Nasdaq stock-months, respectively. First order autocorrelation is significantly different from its

[5] As noted above, returns were measured using mid-spread prices and, therefore, negative first order autocorrelation cannot be attributed to a bid-ask bounce.

benchmark of 0.0 for only 3% and 1% of NYSE and Nasdaq stocks, respectively, while VR is significantly greater than its benchmark of 1.0 for 73% and 81% of NYSE and Nasdaq stocks, respectively. For stocks for which first order autocorrelation is positive, 67% and 72% have VR greater than 1.00 for NYSE and Nasdaq stock-months, respectively. For stocks for which first order autocorrelation is negative, 73% and 82% have VR greater than 1.00 for NYSE and Nasdaq, respectively. Most interestingly, for all stocks and months, as reported in Table 2, the correlation between the variance ratio (VR) and the first order autocorrelation (CORR) of half-hour returns is -0.02 and -0.08 for NYSE and Nasdaq stocks, respectively.

These contrasts suggest that VR and first order autocorrelation are distinctly different measures even though first order autocorrelation is one of the components of VR, and they suggest that higher orders of autocorrelation have an appreciable impact on the dynamic behavior of price formation and, therefore, should be taken account of.

As noted, further insights concerning the presence of price discovery noise can be gained by correlating VR with a set of independent variables that we expect would be more closely related to price discovery noise than to the other sources of intraday noise. Accordingly, significant regression parameters for our independent variables with the correct signs would be evidence that price discovery noise is indeed an important factor. In analyzing this, we focus on VR not CORR because VR is more robust, the reason being that it takes higher orders of autocorrelation into account.

Table 3 shows the average of the monthly regression coefficients for the determinants of VR for the NYSE and Nasdaq. Within each exchange, we report two sample periods: the earlier period (1993-2006) and the later period (2007-2021), as major market structure and regulatory changes have occurred that substantially differentiate the two sub-periods of our sample.[6] The t-statistics computed with Newey-West standard errors are in parentheses.

Market Capitalization (*LNME*): We expect VR to be negatively related to market cap. For both sample windows, LNME is negatively related to VR for Nasdaq stocks and for NYSE stocks for the earlier sample, while it is insignificantly positively related for the later sample.

---

[6] These changes include the NYSE's and Nasdaq's transition from membership organizations to for-profit companies, the introduction of electronic trading, the advent of high frequency and algo trading, Nasdaq's transition from a quote driven dealer market to an order driven electronic limit order book market that more closely resembles the NYSE market, the growth of exchange traded funds, a major loss of market share for both of the major markets, and the NYSE's and Nasdaq's introduction of electronic call auctions to open and close their continuous trading sessions.

Market Share (*NYSE*, *Nasdaq*): We expect VR to be negatively related to market share and it is for both NYSE sample periods (although the coefficient for the latter period is insignificant), but Nasdaq, unexpectedly, has a significantly positive relationship for both sample periods.

First Half-Hour Volume Relative to Daily Volume (*1st HH*): We expect VR to be negatively related to 1st *HH* and it is for both Nasdaq sample periods and for the earlier sample period for NYSE stocks, but not for the later period.

Effective Spread (*Spread*): We expect VR to be positively related to spread and, for both exchanges and both sample periods, it is positive.

Volatility: We expect VR to be positively related to volatility and, for both NYSE samples and the earlier Nasdaq sample it is positive.[7]

In summary, out of twenty coefficients (five variables, two exchanges, and two sample periods), fifteen regression coefficients had the expected sign and most are statistically significant. This suggests that price discovery noise has a meaningful impact on price formation.

Intraday data, including every bid and ask quote, transaction price, and trading volume, became available electronically with the advent of electronic trading. But a lot of factors that are at play on an intraday basis (e.g., bid-ask spreads, market impact, liquidity trading, non-synchronous trading and price discovery noise) make straightforward analyses of price discovery a difficult challenge. With respect to intraday analysis, we find that the variance ratio (VR) is predominantly greater than its neutral value of one, indicating an accentuation of intraday volatility, higher orders of autocorrelation in intra-day returns, and that the EMH is not confirmed for intra-day returns. Further, the multi-variate regression analysis suggests that dynamic price discovery in good part underlies the accentuation of intra-day volatility,

Section 3: Trading, Behavioral Economics and Technical Analysis

Behavioral analysis could help to provide a better understanding of the noisy, dynamic process of price formation in a divergent expectations, adaptive valuations environment. Many individuals make their buy/sell decisions not only according to their own private assessments of

[7] ~~Hua and Schwartz (2024) suggest that volatility being positively related to~~ Hua and Schwartz (2024) suggest that VR being positively related to the spread results from spreads being wider when information is more complex and expectations, therefore, being more divergent. In this uncertain setting, investors are likely to post their quotes less aggressively (i.e., ask quotes are raised and bid quotes are lowered).

share values but also with regard to the assessments of others. As noted in the introduction, information about the assessments of others can be obtained in a number of ways: via the prices that are set in the market (as we take them to be in this paper), through person-to-person discussions, through the advice given by brokers, by public news venues (newspapers, radio and television) and by social media. The complexities of the information transferred and the myriad ways in which it is transferred, can result in participants having behavioral responses as distinct from purely rational responses.[8] Accordingly, behavioral reactions can play an important role in understanding the complex and dynamic process of price formation in financial markets. This analysis interfaces with fundamental and technical analysis by providing insights into the psychological and behavioral factors that influence market participants' decision-making processes. The complexities of information transfer, the reality of investors making fuzzy valuations, and the myriad ways in which information is disseminated can result in participants exhibiting behavioral responses as distinct from rational responses.

Behavioral analysis recognizes that individuals' decision-making processes are influenced by cognitive biases, emotions, and heuristics. Behavioral analysis aims to understand these psychological and behavioral factors, such as herd behavior, overconfidence, loss aversion, and anchoring bias, and their impact on market dynamics. While rational models and technical analysis focus on quantitative data and price patterns, behavioral analysis acknowledges the role of human behavior, emotions, and cognitive biases in shaping market outcomes. In so doing, decisions are influenced by various psychological and social factors and market participants do not always act rationally.

One example of investors behavioral biases affecting price formation is overreaction in the stock market. De Bondt and Thaler (1985) found that investors tend to overreact to unexpected and dramatic news events, causing substantial mispricing of securities that is eventually corrected. They attribute this overreaction to cognitive biases like representativeness heuristic, where investors form overly specific expectations by overweighting recent events, and overconfidence, where investors overestimate the precision of their knowledge about a company's future prospects.

[8] In our own work, however, we have stayed with rational responses and have considered only one way in which information about the market's collective opinion can be disseminated: the prices that are set by traders in the marketplace when they agree to disagree.

As a result, stock prices temporarily diverge from fundamental values until the overreaction is eventually corrected by the market.

Cornell (2018) criticizes the EMH assumption that investors are rational utility maximizers under all states and for all investors. Instead, he suggests that behavioral biases, which are often state-dependent, could serve as a more realistic approach to understanding market anomalies.

Behavioral analysis complements technical analysis by providing insights into the psychological and behavioral factors that influence market participants' decision-making processes that are part of the complexities of fundamental information and price formation. By considering both rational and behavioral responses, researchers and practitioners can gain a more comprehensive understanding of market dynamics and price formation.

Technical analysis is being used by equity market participants to price and to time their orders, and its prevalent use during a trading day is apparent in statements made by both traders and the financial media (to wit, the attention paid to the importance of prices piercing support and resistance levels). Furthermore, with the advent of intra-day algorithmic trading, technical analysis that was historically carried out by human traders now has an expanded use in the world of electronic trading.

Many academics, on the other hand, have considered technical analysis worthless and equivalent to snake oil. The Efficient Market Hypothesis (EMH) states that stock price changes are random (unpredictable) and cannot be used to predict future prices. Technical analysis would indeed have no value if the prices of equity shares followed random walks and markets were informationally efficient.

Strong opinions have been expressed about technical analysis and the EMH. In *A Random Walk Down Wall Street*, Burton Malkiel (1973) wrote, "Technical strategies are usually amusing, often comforting, but of no real value" Further, he writes, "On close examination, technicians are often seen with holes in their shoes and frayed shirt collars. I have personally never known a successful technician, but I have seen the wrecks of several unsuccessful ones." He continues, "The past history of stock prices cannot be used to predict the future in any meaningful way." Malkiel would be correct if markets were informationally efficient.

On the other side of the spectrum, Robert Schiller (1984) called the efficient market hypothesis "one of the most remarkable errors" in the history of economics, and Lawrence

Summers (1986) wrote that the "Efficient Markets Hypothesis is a shared act of faith, with little in the way of theoretical or empirical support." What should be made of this divergence of opinions?

As we have noted, markets would indeed be informationally efficient if they were frictionless and participant expectations were homogeneous. However, neither applies in reality. Bernard Baruch's (1957) description of the marketplace provides a good foundation for focusing on investor expectations being divergent:

> The prices of stocks – and commodities and bonds as well – are affected by literally anything and everything that happens in our world, from new inventions and the changing value of the dollar to vagaries of the weather and the threat of war or the prospect of peace. But these happenings do not make themselves felt in Wall Street in an impersonal way, like so many jigglings on a seismograph. What registers in the stock market's fluctuations are not the events themselves but the human reactions to these events, how millions of individual men and women feel these happenings may affect the future.

Would millions of individuals have identical interpretations of these happenings? Would these expectations affect the future to be homogeneous? Edward Miller (1977) provides an answer:

> …it is implausible to assume that although the future is very uncertain, and the forecasts are very difficult to make, that somehow everyone makes identical estimates of the return and risk from every security. In practice, the very concept of uncertainty implies that reasonable men may differ in their forecasts.

Two best-seller finance textbooks we reviewed in the areas of Corporate Finance[9] and Investments[10], on the other hand, are by and large, supportive of the EMH, a stance that is in contrast to technical analysis and various computerized trading algorithms that rely on past price moves to make trading decisions[11]. The textbooks do include, however, the caveat that the efficient markets hypothesis is an ongoing debate, and there are mixed views regarding its validity.

---

[9] Fundamentals of Corporate Finance, 13th edition, 2022, authors Ross, Westerfield, and Jordan.
[10] Investments, 12th Edition, 2022, authors Bodie, Kane and Marcus.
[11]This includes algorithmic trading that is driven by artificial intelligence and machine learning models.

As described in Section 2, overshooting and reversal behavior can occur in a divergent expectation, adaptive valuations environment. Accentuated intra-day volatility and its counterpart, returns autocorrelations of first and higher orders, contradicts the EMH and opens the door for technical analysis to play a role in price discovery.

Technical analysis, however, is not simple. The dynamic behavior of prices is complex and constantly subject to change. Successful patterns can be picked up and either changed or eliminated by more participants exploiting them, only to be replaced by other patterns. Furthermore, the patterns that technical analysis might seek to exploit are not confined to first order autocorrelation but, as our variance analysis shows, higher orders of autocorrelation are also important. Moreover, the patterns in intra-day returns can be quite different than those observed in inter-day returns. Nevertheless, technical analysis is being widely used by market participants, especially on an intra-day basis.

In a non-frictionless, divergent expectations environment, equilibrium prices are not instantly achieved, but are gradually reached as trading proceeds following the arrival of new information. Trading, of course, requires that orders be submitted by both buyers and sellers. An equivalent buying and selling of individual stocks will not occur in a CAPM world because the optimal combination of stocks in the market portfolio is the same for all participants. In CAPM, for their own liquidity needs and/or changing tastes for risk, some participants seek to lessen their holdings of cash in order to increase their holdings of the market portfolio, while others seek to lessen their holdings of the market portfolio in order to increase their holdings of cash. In contrast, in a divergent expectations environment, trading and price discovery occur on the individual stock level as well as on the aggregate level.

In short, to understand trading and the complexity of price discovery, a key CAPM assumption must be relaxed: that informed participants have homogeneous expectations. Much trading is no doubt motivated by investors having divergent expectations with bullish participants coming to the market as buyers of specific stocks, and bearish participants coming to the market as sellers.[12] And, when participant expectations are divergent, participants looking to trade for their own idiosyncratic cash flow (liquidity) needs are not required for a market to operate. That is, when their expectations are divergent, informed traders will trade with each other, and they,

[12] There are also those traders who trade for arbitrage or hedging purposes.

without liquidity traders, can support a market that is replete with runs and reversals. Noise traders are also present in the form of technical analysts.

Section 4: The Impact of Market Structure

In a homogeneous expectations environment, share values are known by informed investors, they are not dependent on market structure, and regulation is called for only with respect to dishonesty and the exploitation of power and position. In contrast, price discovery in a divergent expectation, adaptive valuations environment is a non-instantaneous, complex process, and the efficiency with which it operates depends on the rules that determine how customer orders are brought together and turned into trades and transaction prices (i.e., a market's architecture). Thus, a major consequence of informed investors having divergent expectations is that market structure matters and regulation has a role to play.

There are many decisions to make with respect to market architecture, and tradeoffs and unintended consequences have to be contended with. With the advent of electronic trading, markets worldwide have become far more efficient as orders are submitted and turned into trades with much faster speeds, and transaction prices are more rapidly displayed. But, has the quality of price discovery been bettered? Have intra-day price volatility and returns autocorrelations attributable to noisy price discovery been improved?

The data, as we have analyzed them, show that price discovery has remained a noisy process. Clearly, market structure, including government regulations that affect market structure, remain works in process. Unfortunately, insufficient attention has been given to the complexity of price discovery that is a property of a divergent expectations, adaptive valuations environment. We present one finding that shows how market structure change can affect the efficiency of price discovery.

Hua and Schwartz (2024) presents a finding that suggests that, in a divergent expectations, adaptive valuation environment, market structure does impact the quality of price discovery. They use the market model to assess how the prices of individual stocks follow the market:

$$r_i = a_i + b_i r_M + e_i$$

where r_i is the return on the i^{th} stock, r_M is the return on the market, b_i is the i^{th} stock's beta coefficient, and e_i is the i^{th} stock's residual error term.

In a frictionless, homogeneous environment, the market model parameters would be independent of the time span over which returns are measured (e.g., daily or monthly). However, it has long been established that the market model beta parameter is, on average, significantly lower when estimated using daily returns instead of monthly returns.[13] Concurrently, the market model's coefficient of determination (R^2) should also be independent of the return measurement interval, but it too is generally lower when daily returns are used instead of monthly returns. Something is going on with respect to the determination of intraday prices, and price discovery appears to be a part of the answer.

Hua and Schwartz (2024) assess market structure's effect on estimates of the market model by focusing on how the market model's R^2 changes when it is obtained using daily returns instead of monthly returns. To this end, they define R^2diff = R^2_{month} - R^2_{day} and regress R^2diff on several causal factors for two separate sample periods (1993-2006 and 2007-2021) and two different markets (the NYSE and Nasdaq). One of the independent variables in their regression analysis reflects both market structure and market structure regulation: the consolidation of order flow (i.e., NYSE's and Nasdaq's market shares). Recognizing that order flow consolidation in an exchange is a major feature of market structure, Hua and Schwartz include NYSE and Nasdaq market share as independent variables in their multi-variant regression analysis.

If geographically consolidating the order flow facilitates price discovery, we would expect R^2diff to be negatively related to the market share variable for each of these two markets. For the 1993-2006 sample, market share's slope coefficient for the NYSE was -0.049 (with a t statistic of -1.80), while, for Nasdaq, it was a considerably different +0.913 (with a t statistic of +3.46). For the 2007-2021 sample, market share's slope coefficient for the NYSE was -0.254 (with a t statistic of -9.00) and its slope coefficient for Nasdaq was -1.54 (with a t statistic of -7.19).

Nasdaq's positive market share coefficient for the earlier period stands in sharp contrast with its negative coefficients in the later period and with the NYSE's negative coefficient in both periods. We suggest that market structure change in good part accounts for this contrast.[14] In the earlier period, the NYSE was an order driven market and Nasdaq was quote driven. In the later

[13] For instance, Scholes and Williams (1977), Cohen et al. (1978), Dimson (1979), and Cohen et al. (1980) have shown that the use of short period (e.g., daily) returns instead of longer period (e.g., monthly) returns to estimate the market model significantly impacts beta coefficients.
[14] Very high market share ratios in the earlier period (roughly 80% for the NYSE and 100% for Nasdaq) no doubt also affect the R^2diff regression parameter because, when they were as high in the earlier sample, the room for variation in the independent variable is sharply curtailed.

period, Nasdaq became order driven and, in so doing, it far more closely resembled the NYSE. In so doing, Nasdaq's response to the market concentration variable became similar to that of the NYSE. The association of Nasdaq's market structure being more like that of the NYSE, with R^2diff's response to the market share variable being more aligned for the two exchanges, underscores the connection between price discovery noise and market structure.

What underlies both the complexity of price discovery and the importance of market structure? Investors having divergent expectations.[15]

Section 5: Conclusion

In CAPM equilibrium, public information is fully reflected in market prices, profitable trading opportunities never exist, share prices follow random walks, the Efficient Markets Hypothesis holds, behavioral analysis does not have a role to play, technical analysis is useless, and issues concerning trading and market structure are not addressed. In a CAPM environment, individual stock price changes that occur following the advent of news do so in some unspecified way that does not include actual trading.

Moreover, because participants value individual stocks identically in a CAPM world, there is no need to discover prices, and the only equity market trading would be exchanging shares of the market portfolio for more or less of the risk-free asset. With the market portfolio being the same for all investors, individual stock trading, price discovery, market structure, and the regulation of market structure are all vacuous.

However, volatility analysis changes the frictionless market picture by revealing the existence of first and higher orders of autocorrelation in short period (intra-day) returns that occurs as prices are being set by the orders of bullish and bearish participants in a divergent expectations environment. We conclude that CAPM, while a strong model, is incomplete.

That being said, the frictionless market analysis of CAPM nevertheless has an important role to play that should be understood along with the workings of a non-frictionless, divergent expectations, adaptive valuations, fuzzy evaluations environment. An analogy suggests why. The Gulf Stream is a deep, strong current that runs from Mexico up the Atlantic Coast and then crosses

[15] A review of regulatory changes in market structure shows that most regulations address the symptoms of price discovery errors, but they do not always focus on improving the price discovery process itself. Schwartz, Ross and Ozenbas (2022) provides a brief historical perspective on the evolution of equity market structure that includes prominent regulatory initiatives.

the Ocean to Europe. This current should be taken account of by ships crossing the Atlantic. But a ship's captain must also deal with the winds, waves, and storms that rile the surface of the sea. Ozenbas, Pagano, Schwartz and Weber (2022) consider the power of the Gulf Stream as analogous to the power of CAPM's determination of underlying, unseen equilibrium values that depend on the means, variances, and covariances of returns. Concurrently, price discovery noise in a divergent expectations environment (along with illiquidity, trading costs, and other impediments of a non-frictionless market) is analogous to the winds, waves, and storms on the surface of the rolling sea. Both the underlying Gulf Stream and the turbulence on the surface of the sea should be understood.

When exchanges and government regulators are considering market structure changes, their fundamental objectives must be articulated. One of the most important objectives should be to improve the quality of price discovery. Thus far, however, price discovery has received insufficient attention. Is it because equilibrium prices are not observable and thus deviations from equilibrium cannot be seen? Is it because investor expectations are too often taken to be homogeneous, and that the implications of divergent expectation have not been adequately understood? We believe that the answer to both questions is "yes." In a world characterized by divergent expectations, adaptive valuations, and fuzzy valuations, more attention needs to be given to price discovery, an all-important function of a stock exchange.

ACKNOWLEDGMENTS
We thank Frank Fabozzi, Lingyi Kong, Michael Pagano, and James Ross for their comments and suggestions.

REFERENCES

Baruch, Bernard M. (1957). *My Own Story*. Henry Holt and Company, New York. Page 84.

Bodie, Z., Kane, A., and Marcus, A. (2022). *Investments, 12th Edition*. McGraw Hill.

Condorcet

Cornell, Bradford. (2018). What Is the Alternative Hypothesis to Market Efficiency?. *Journal of Portfolio Management* 44 (7): 3–6.

Davis, P and Schwartz, R.A. (2021). Plato or Aristotle: Who Got It Right? Evidence from the Equity Markets, *Journal of Portfolio Management,* 47(3), 1-8.

Estlund, D. M. (1994). Opinion leaders, independence, and Condorcet's jury theorem. *Theory and Decision*, *36*(2), 131–162.

Hua, J. and Schwartz, R.A. (2024), Price discovery noise, working paper.

Ladha

Lewellen, J. (2022), Autocorrelation of stock and bond returns, 1960–2019, working paper.

Malkiel, B. G. (1973). *A random walk down wall street*. W.W. Norton & Company.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *Journal of Finance*, 32(4), 1151–1168.

Ozenbas, D., Pagano, M. S., Schwartz, R. A., and Weber, B. W. (2022). *Liquidity, markets and trading in action: An interdisciplinary perspective*. Springer.

Prato, C., and Wolton, S. (2022). Wisdom of the crowd? information aggregation in representative democracy. *Games and Economic Behavior,* 135, 86–95.

Handa, P., Schwartz, R.A., and Tiwari, A. (2003). Quote setting and price formation in an order driven market. *Journal of Financial Markets,* 6(4), 461–489.

Ross, Stephen, Randolph Westerfield, and Bradford Jordan. (2024). *Fundamentals of Corporate Finance, 13th Edition*. McGraw Hill.

Shiller, R. (1984). Stock prices and social dynamics. *Brookings Papers on Economic Activity,* 2, 457–498.

Schwartz, R.A., Ross, J. and Ozenbas, D. (2022), Equity Market Structure and the Persistence of Unsolved Problems: A Microstructure Perspective, *Journal of Portfolio Management,* special issue.

Schwartz, R.A., Wolf, A. and Paroush, J. (2010), The dynamic process of price discovery in an equity market, *Managerial Finance*, 36(7), 554-565.

Summers, Lawrence H. (1986) Does the Stock Market Rationally Reflect Fundamental Values? *The Journal of Finance,* vol. 41, no. 3, pp. 591–601.

Surowiecki, J. (2005). *The Wisdom of Crowds*. Anchor Books.

De Bondt, W.F. and Thaler, R., 1985. Does the stock market overreact?. *The Journal of finance*, *40*(3), pp.793-805.

Table 1 Summary statistics

This table reports the time-series averages of the cross-sectional mean, median (P50), standard deviation, skewness, excess kurtosis, and 25th and 75th percentile of the main variables used in this paper. All variables are computed for individual firms at the end of month t. VR is the ratio of annualized standard deviation of intraday 30-minute returns over the monthly return standard deviation. CORR is the daily return autocorrelation in the month. Other stock specific variables include: the log of market cap at the end of month (LNME), the monthly average of the proportion of the first half hour shares to the daily number of shares (1st HH share), the monthly average of daily equally weighted effective spreads obtained from WRDS's intraday indicators (Spread), the daily return standard deviations in a monthly as Volatility. Panel A reports statistics for all NYSE stocks, and it also reports the NYSE market share (NYSE share). Panel B reports statistics for all Nasdaq stocks and the corresponding market share (Nasdaq share). The sample covers the period from January 1993 to December 2021.

Panel A: NYSE stocks

						1993-2006		2007-2021	
	Mean	P50	Std	P25	P75	Mean	Std	Mean	Std
VR	1.29	0.96	2.14	0.77	1.24	1.14	0.84	1.43	3.36
CORR	-0.06	-0.07	0.23	-0.22	0.09	-0.07	0.24	-0.06	0.22
LNME	13.84	13.78	1.82	12.54	15.03	13.4	1.8	14.24	1.83
NYSE	53.60%	54.87%	9.42%	48.85%	59.64%	82.56%	12.07%	26.58%	6.92%
1st HH	7.00%	8.12%	5.22%	1.59%	10.70%	7.52%	6.08%	6.58%	4.42%
Spread	0.37%	0.20%	0.58%	0.12%	0.37%	0.62%	0.95%	0.14%	0.24%
Volatility	2.14%	1.81%	1.53%	1.25%	2.60%	2.06%	1.51%	2.22%	1.55%
Turn	3.23%	0.06%	41.71%	-21.40%	23.85%	5.48%	45.92%	1.14%	37.78%

Panel B: Nasdaq stocks

						1993-2006		2007-2021	
	Mean	P50	Std	P25	P75	Mean	Std	Mean	Std
VR	1.75	1.21	2.25	0.89	1.84	1.72	1.54	1.78	2.86
CORR	-0.14	-0.15	0.24	-0.31	0.03	-0.18	0.25	-0.11	0.23
LNME	11.97	11.91	1.82	10.7	13.16	11.34	1.71	12.56	1.93
Nasdaq	60.42%	60.32%	7.25%	55.63%	65.01%	90.41%	4.26%	32.42%	10.03%
1st HH	26.36%	10.01%	789.25%	7.18%	13.54%	10.50%	6.70%	41.17%	91.64%
Spread	2.01%	1.23%	2.21%	0.64%	2.60%	3.06%	2.79%	1.03%	1.66%
Volatility	3.83%	3.08%	3.24%	2.10%	4.58%	4.42%	3.61%	3.28%	2.90%
Turn	3.87%	-1.97%	68.28%	-36.15%	36.52%	4.62%	71.32%	3.16%	65.43%

Table 2 Correlation

This table reports the times-series averages of the monthly cross-sectional correlations between all the variables in the paper. Panel A reports statistics for all NYSE stocks, and Panel B reports statistics for all Nasdaq stocks. The sample covers the period from January 1993 to December 2021.

Panel A: NYSE stocks

	VR	CORR	LNME	NYSE	1st HH	Spread	Volatility
CORR	-0.02						
LNME	-0.09	0.07					
NYSE	-0.06	0.02	0.00				
1st HH	-0.02	0.04	0.20	0.01			
Spread	0.19	-0.07	-0.59	-0.17	0.17		
Volatility	0.15	0.03	-0.19	-0.16	0.25	0.45	
Turn	0.14	0.09	-0.01	-0.11	0.06	-0.03	0.29

Panel B: Nasdaq stocks

	VR	CORR	LNME	Nasdaq	1st HH	Spread	Volatility
CORR	-0.08						
LNME	-0.26	0.22					
Nasdaq	0.09	-0.07	0.01				
1st HH	0.08	-0.10	-0.21	0.01			
Spread	0.35	-0.26	-0.66	0.05	0.34		
Volatility	0.17	-0.01	-0.35	-0.08	0.14	0.40	
Turn	0.13	0.13	0.00	-0.08	0.01	-0.05	0.30

Table 3: Intraday/daily analysis: Determinants of VR

This table reports the average slope coefficients for VR regressed on a set of contemporaneous variables using Fama-MacBeth (1973) methodology. See Table 1 for the definitions of all variables. N is the total number of observations in the regression, and R-sq is the average R-squared of the monthly cross-sectional regressions. Newey-West t-statistics are reported in parentheses.

	NYSE			Nasdaq	
	1993-2006	2007-2021		1993-2006	2007-2021
VR	(1)	(2)		(3)	(4)
LNME	-0.050	0.177		-0.070	-0.108
	[-8.48]	[5.16]		[-3.17]	[-11.00]
NYSE	-0.178	-0.291		18.530	2.262
	[-3.07]	[-0.65]		[3.78]	[5.29]
1st HH	-1.840	1.376		-1.720	-0.583
	[-9.86]	[2.39]		[-7.17]	[-1.82]
Spread	0.124	0.147		0.284	0.529
	[12.12]	[1.86]		[13.76]	[5.69]
Volatility	6.509	1.034		0.449	-2.128
	[10.79]	[0.60]		[0.15]	[-1.78]
Intercept	1.900	-1.322		-16.100	2.283
	[11.75]	[-3.06]		[-3.16]	[9.43]
N	246558	215023		507425	362787
R-sq	0.174	0.087		0.242	0.149